Orthogonal Determinants

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Abstract. Basic concepts and notions of orthogonal representations are introduced. If $\mathfrak{X} : G \to \mathrm{GL}(V)$ is a K-representation of a finite group G it may happen that its image $\mathfrak{X}(G)$ fixes a non-degenerate quadratic form q on V. In this case $\mathfrak{X}$ and its character $\chi : G \to K$, $g \mapsto \mathrm{trace}(\mathfrak{X}(g))$ are called orthogonal. If χ is an irreducible orthogonal character of even degree this form is unique up to scalars and there is a unique square class $\det_\chi$ in the character field $\mathbb{Q}(\chi) = \mathbb{Q}(\chi(g) \mid g \in G)$ such that given any field L with a representation that affords χ, the determinant of the fixed form q is $\det(q) = \det_\chi(L^\times)^2$. In this talk we will discuss computational methods for determining this square class which is called the orthogonal determinant of χ. We will also briefly mention theoretical tools for determining this square class.